NOYES ANPA 9 PROCEEDINGS, SEPT. 198

QUERY 5: Can you, by using the relativistic discrete theory... obtain the Bohr-Sommerfeld fine structure splitting for the hydrogen spectrum...

McGOVERAN, June 24, 1988

THEOREM: YES

There once was a string theorist,
Who thought things up to sew,
But on close inspection
His fundamental material
Was only GUT and not TOE!

THE FINE STRUCTURE OF HYDROGEN (and related results)

SYSTEM DEFINITION

- ORDERING OPERATOR CALCULUS
  - DISCRETE, FINITE, MULTIPLE CONNECTIONS,
  - HAMMING DISTANCES, NO SQUARE ROOTS
- THE COMBINATORIAL HIERARCHY
  - $3, 7, 127, 2^{127}-1$
- PROGRAM UNIVERSE
  - GLOBAL ORDERING OPERATOR
  - $\beta = \frac{2k}{n} - 1$, EVENT, VERTICES,
  - COULOMB EVENT PROBABILITY = $1/137$
- 2 INDEPENDENT PERIODS J, S
- 1 DEPENDENT PERIOD R
  - Characterizes system
- BASED ON LEVEL 3 EVENTS

BIT STRING PROBABILITIES

- FOR STRINGS of length L bits,

  let $L_r = L_{s_0} + 1$ AND

  $$s_{n+1} = s_n + 1 \; ; \; j_{n+1} = j_n + 1$$

  AND define r by

  $$j_0^2 = s_0^2 + r^2 \qquad (I)$$

- TO REPRESENT 127 strings AT level 3,

  $L_r = 7$

  so

  $L_s = 6$

- "HISTORY": WAYS to generate strings of k one's of length $L = n$ is

  $$\sum_{k=0}^{n} \binom{n}{k}$$

- FIX k even ; $k \neq 0, n$

- Ways to generate S are then

$$\sum_{k=1}^{2} \binom{6}{2k} = 30$$

- Ways to generate an S, j pair

$$30 \times 127$$

- Probability that an S, j pair will <u>not</u> be generated

$$P(\overline{sj}) = 1 - \frac{1}{30 \times 127} \qquad \text{(II)}$$

- S can be "phase-shifted" with respect to j in two ways:

  - $137 j_0 + 137 s_0 = b_+ \;,\; j_0 + s_0 = \frac{b_+}{137} \qquad \text{(III)}$
  - $137 j_0 - 137 s_0 = b_- \;,\; j_0 - s_0 = \frac{b_-}{137} \qquad \text{(IV)}$

- LET

$$a^2 = \frac{(1+\epsilon)(1-\epsilon)}{(137)^2} = \frac{(b_+)(b_-)}{(137)^2} = j_0^2 - s_0^2 \quad \text{(V}$$

- THEN, for integer n and definition of s_i, j_i

$$s = n + s_0 = n + \sqrt{j^2 - a^2} \qquad (\mathrm{VI})$$

CAUTION: Note square root!

- LET H' be value corresponding to H, the energy attribute of the system for a Coulomb event, 137_j and mc^2 the total energy. The "phase shift" energy due to s_0 is just:

$$(a/s)(H')$$

so that

$$(H')^2 + \left[\left(\frac{a}{s}\right) H'\right]^2 = (mc^2)^2$$

- Rearranging and taking (VI) into account, we obtain

$$\frac{H'}{mc^2} = \left[1 + \frac{a^2}{\left(n + \sqrt{j^2 - a^2}\right)^2}\right]^{-1/2} \qquad \text{(VII)}$$

THE BOHR-SOMMERFELD FORMULA

- Given the physical situation and the definition of a^2, we can only use the factor $1-\epsilon$ to define a, the factor $1+\epsilon$ being s in the subsequent period of j.

Thus

$$a^2 = (1-\epsilon)^2 / 137^2$$

and

$$a = (1-\epsilon)/137 \qquad \text{(VIII)}$$

- By construction

$$\epsilon = P(s_j) = \frac{1}{30 \times 127}$$

- Substituting we have $a = \left(1 - \frac{1}{30\times127}\right)/137$

$$a^{-1} = 137.0359674\ldots$$

As compared to empirical fine structure constant

$$\alpha^{-1} = 137.035963(15)\ldots$$

and so QED.